

Optimal Sizing and Allocation of Residential Photovoltaic Panels in a Distribution Network for Ancillary Services Application

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Abstract- Tremendous penetration of renewable energy in electric networks, despite its valuable opportunities, such as balancing reserve and ancillary service, has raised concerns for network operators. Such concern stems from grid operating conditions. Such huge penetration can lead to violation in the grid requirements, such as voltage and current limits. This paper proposes an optimization method for determining the number of photovoltaic (PV) panels together with their arrangement in the grid in order to maximize ancillary service, without violating grid operation limits. The proposed optimization method is based on genetic algorithm. To do so, single-objective and multi-objective optimization have been considered. The proposed method is implemented on a model of a part of a Danish distribution grid to verify its effectiveness. The simulation results prove the viability of the method, while keeping the grid requirements within standard operating limits.

I. INTRODUCTION

While government policies and renewable energy initiatives are great incentives to increase small PV installation for the next decades [1], it has raised technical concerns regarding voltage rise and reverse power flow in distribution networks. Overvoltage problem commonly has been reported in the distribution networks with high PV penetration level when a considerable number of households decided to install PV panels [2]-[5]. To overcome this problem, different methods, such as reactive power control methods among feeders and imposing curtailment, are proposed to deal with the overvoltage issue [6]. Additionally, as PV penetration increases, more reverse power flow will be directed toward upper grid, which consequently leads to a new set of technical issues. Such reverse power flow might exceed voltage limitations and power limitations of transformers. However, the reverse power flow can be notified as a source of ancillary services for the upper grid. In this paradigm, an excess PV generation might be purchased by the system operators instead of conventional spinning and non-spinning reserves. This might be very helpful for the system operators to reduce costs and transmission congestion,

particularly during peak hours. As a result, larger PV penetration essentially provides more ancillary service resources for the upper grid, considering the fact that the technical limitations of the distribution network should not be violated.

Different methods for sizing and allocation of distributed generation in the distribution networks are proposed in different studies [7]-[9]. However, these studies focus on minimizing operational cost and/or emission, loss minimization [8], and voltage correction and so on.

In this paper, maximum PV penetration is formulated as an optimization problem to maximize reverse power flow to the upper grid, as a source of ancillary service. To the best of our knowledge, this objective for residential PV sizing and allocation has not been reported in any literature. Different objectives are defined and compared (as single and multi-objective) for PV sizing and allocation for a sample distribution network. Genetic algorithm (GA), as a heuristic optimization technique, is employed to solve the nonlinear optimization problem. In order to follow practical application of residential PV panels, the PV sizing problem is formulated as an integer optimization problem. Therefore, the number of PV panels installed on each bus is limited to the number of households on the same bus. Different technical constraints, such as voltage and current limits of lines, buses, and the transformer nominal power, are considered in the optimization problem. Finally, the proposed method is utilized for a sample distribution network, where size and location of PV panels is calculated on each bus for different objective functions. The rest of the paper is organized as follows. The optimization problem is explained in section II. Also, different objective functions, as well as their coefficients, are explained in detail in this section. The applied optimization technique is presented in section III. The proposed method is tested on a model of a Danish distribution grid. Section IV presents the grid overall view. Section V discusses optimization results and compares the outcome of different optimization methods. The conclusion of the results

is presented in section VI.

II. METHODOLOGY

It is well-known that ancillary service is a major concern in power system operation, where generation and demand should always remain balanced within power systems. Traditionally, spinning and non-spinning reserves were the only options to provide ancillary services. However, the large PV penetration level in the distribution systems provided a new opportunity for ancillary services which possibly can decrease operational cost and transmission congestion [10]. Because of the paradigm shift in ancillary services, maximum PV penetration in the distribution networks is desirable for the system operators, as long as it does not violate any technical limitations. To do so, a general approach is proposed to determine the maximum PV size and locations in a distribution network for maximum grid support without violating any technical constraint. In this study, three different objectives are considered which are structured as different single and multi-objective functions. The proposed objectives include: 1) maximizing power to the upper grid (reserve), 2) minimizing voltage deviation, and 3) minimizing power loss. Because grid support is the primary goal of this study, the main objective is the first objective. Considering different grid requirements and their correlation, such as the correlation between power and voltage or power and losses, the optimization problem is complex. Due to the correlation between objectives, optimizing one function might lead to unwanted effects on other objectives, which might not be desirable. Therefore, simultaneous optimization of the objective functions can lead to better results, while grid constraints must also be met [7], [11]. Therefore, two methods can be defined to deal with such problem: “single-objective method” and “multi-objective method”. Single and multi-objective formulations are explained in the following sub-sections.

A. Single-Objective Approach

In general, the single-objective method can be expressed as:

$$\begin{aligned} & \min f(x) \\ & \text{subject to: } \begin{cases} g_i(x) = 0 ; & i = 1, 2, \dots, m \\ h_j(x) \leq 0 ; & j = 1, 2, \dots, n \end{cases} \end{aligned} \quad (1)$$

Where m is the number of equality constraints and n is the number of inequality constraints. As mentioned before, different functions can be defined for this problem, with respect to what the user expects from the optimization. For our purpose, maximizing “power to the upper network” (called $P_{upper}(x)$) is a priority. Therefore, the optimization problem with a single objective function is limited to maximize power to support the upper grid. Based on the convention used in this study, power flow to the upper grid is negative. As a result, the problem of maximizing power flow to the upper grid will alternatively be changed to minimizing power flow through transformer, as follows:

$$\max P_{upper}(x) \square \min (\text{mean}(P_{trafo}(x))) \quad (2)$$

In (2), the average power flowing through the transformer to the feeders during the day is obtained. Then, to maximize the power to the upper grid, this average value is minimized.

In this study, no equality constraint is involved in the optimization problem. Inequality constraints are similar to the alternative problem where transformer nominal capacity, standard voltage deviation for buses, and cable nominal currents are considered, i.e.: 1) Voltage of all buses, $V_{b,s}$, must stay within a specific limit, i.e., $V_{min} \leq V_{b,s} \leq V_{max}$; $b = 1, 2, \dots, l$. 2) Power flowing through transformer must be less than transformer nominal power. 3) Current limits of the cables. Here, V_{min} and V_{max} are the minimum and maximum voltage values of the grid buses, b is the number of the bus, and S is the time interval of the calculation. In this work, 15-minute time intervals are considered for calculations.

B. Multi-Objective Approach

Although the priority of the PV sizing and allocation optimization is given to the grid support objective, it is desired to consider other technical issues in the optimization simultaneously. Therefore, the multi-objective (MO) optimal structure is proposed as [12]:

$$\begin{aligned} & \min F(x) \\ & F(x) = [f_1(x), f_2(x), \dots, f_k(x)] \end{aligned} \quad (3)$$

From (3), k different functions (namely $f_k(x)$) might be considered in the MO problem. Similar constraints as in the single-objective function problem are utilized for the MO problem as well. Here, K is the number of objective functions. In our study, three objective functions are considered, as follows:

$$f_1(x) = \max P_{upper}(x) \square \min (\text{mean}(P_{trafo}(x))) \quad (4)$$

$$f_2(x) = \max(\max(|V_{b,s} - 1|)) \quad (5)$$

$$f_3(x) = P_{loss, total} \quad (6)$$

Where $V_{b,s}$ is the voltage at bus b at time interval s . As can be seen, the function defined in (4) is similar to that in (2). In (5), maximum voltage deviation at each grid bus is calculated during the day. Among all these voltage deviations, the maximum deviation is taken into account for optimization, where the optimization procedure according to (3) tries to minimize this maximum deviation. Also, the grid total loss during the day is calculated and taken as the third objective to be minimized, as mentioned in (6). Similar inequality constraints given for the single-objective optimization problem in (1) are utilized for the MO problem as well. As mentioned above, MO function can be obtained by combining different objectives. Different methods have been presented so far to deal with MO problems, among which Pareto optimality is a predominant method [12]. A comprehensive review of different Pareto optimality methods

is presented in [12]. A very common method for combining different objective functions, which is used in this paper, is called “weighted-sum approach”. In this method, different objective functions are added together with a weighting factor to form a single objective function as given below:

$$\min \sum_{i=1}^k w_i f_i(x) \quad (7)$$

Objective functions are weighted based on their significance in the optimization. Typically, sum of all weighting factors are equal to one:

$$\sum_{i=1}^k w_i = 1 \quad (8)$$

Where $f_i(x)$ is i^{th} objective function from k existing objectives. In this method, similar constraints can be employed for the optimization problem in hand. Typically, there are two methods to calculate weight factors: “rank order centroid” (ROC) method, and “rank sum” (RS) method [11].

ROC method: In this method, a uniform distribution of the weights is assumed on the simplex of rank-order weights.

Then, for $w_1 > w_2 > \dots > w_m$, if i is the rank position of w_i , and k is the number of objective functions:

$$w_i = \frac{1}{k} \sum_{l=i}^k \frac{1}{l} \quad (9)$$

RS method: This method is appropriate when there is a priority sequence among the objective functions. The weighting values can be calculated as follows:

$$w_i = \frac{2(k+1-i)}{k(k+1)}, \quad i = 1, 2, \dots, k \quad (10)$$

As mentioned earlier, the highest priority is given to the first objective. Voltage deviation and power loss are second and third in the list, respectively. Minimizing voltage deviation has higher priority than power loss, since power loss does not change significantly with different PV sizes and allocation. This is due to the grid size under study and its low power loss. This fact will be shown in the section V.B as a simulated case study. The focus of this paper is to maximize power flow to the upper grid. Optimization variables are the number of PV panels for each bus in the grid. To set the upper limit for the variables, it is assumed that a typical household installs a maximum of 6-kW (2 sets of 3-kW) PV panel. Such assumption sounds reasonable, considering the available roof area of a typical house.

III. OPTIMIZATION TECHNIQUE

With the objective functions and constraints in hand, it is required to utilize an appropriate optimization technique to solve the problem. The optimization problem in this study includes integer decision variables, namely the number of PV panels which can be installed on each bus. In addition, the objective functions (whether single objective or MO problems) are nonlinear functions, and might not converge.

Therefore, heuristic optimization methods seem to be the best option for the problem in hand. There are several heuristic optimization techniques with their own weaknesses and strengths. However, only a few of them are capable of solving integer optimization problems. Amongst all, (GA) is chosen in this study to solve the optimization problem. It is available in a MATLAB optimization toolbox, and is capable of solving integer optimization problems. Genetic algorithm is a search heuristic that imitates the process of natural selection by routinely providing solutions to optimization problems. Some main parameters of this method are: selection, crossover, mutation, and population. The principles of this technique, together with its different parameters are explained in [13]-[15]. Details of GA parameters are presented in Table I. For the MO problem, different weighting factors are calculated based on (9) and(10), and is reported in Table II. As an example, to calculate the weight factor for the first objective function, $f_1(x)$, in ROC method, we would have:

$$i = 1 \Rightarrow w_1 = \frac{1}{3} \sum_{l=1}^3 \frac{1}{l} = \frac{1}{3} \left(1 + \frac{1}{2} + \frac{1}{3} \right) = \frac{11}{18}$$

For $f_2(x)$, the calculation would be similar:

$$i = 2 \Rightarrow w_2 = \frac{1}{3} \sum_{l=2}^3 \frac{1}{l} = \frac{1}{3} \left(\frac{1}{2} + \frac{1}{3} \right) = \frac{5}{18}$$

The same calculation can be done for the rest of the factors.

TABLE I. GA PARAMETERS.

Name	Population	No. of generation	Stall generation	Tolerance function
Value	40	200	100	1e-8

TABLE II. WEIGHTING FACTORS FOR DIFFERENT OBJECTIVES IN THE MO PROBLEM.

Method	Objective Function		
	$f_1(x)$	$f_2(x)$	$f_3(x)$
ROC	11/18	5/18	2/18
RS	1/2	1/3	1/6

IV. CASE STUDY

In order to evaluate the effectiveness of the proposed PV sizing and allocation method, several simulation studies are carried out on a distribution network model. The distribution network model is chosen from Danish distribution grid, and its single line diagram is shown in Fig.1. The number of households on each feeder is presented in Table III. All the grid buses are shown in this figure. However, to keep the figure more obvious, only some of the households are shown in the figure. In order to build a practical distribution network for simulation studies, appropriate load data and PV model are utilized. Considering available grid data, the “Velander method”[16] is applied to determine load demand. This method is created based on large empirical data and is widely accepted among distribution system operators (DSOs) and researchers in Scandinavian countries. In this study, the actual load demand data for a typical summer day is used which

originally was available as annual energy demand data. Eq (11) represents the Velander formula [16].

$$P_{\max} = 0.00033 * E + 0.05 * \sqrt{E} \quad (11)$$

In this equation, E is the annual energy demand of the household. To have a more realistic load model, “coincident load behavior” is also taken into account [16]. Eq (12) presents the formula for calculating this factor.

$$cl = \frac{P_{\max}}{\sum_{i=1}^n P_{\max i}} \leq 1 \quad (12)$$

In this equation, n is the number of households on a feeder, P_{\max} is the maximum demand of the feeder, and $P_{\max i}$ is the maximum demand of household ‘ i ’ on the feeder. For residential solar PV panels, a realistic and simplified model is used in this study [17], [18]. Such model gives accurate results, while eliminating unnecessary details for a steady state analysis. Eq (13) describes the model.

$$P_{pv} = \frac{G}{1000} * P_{STC} [1 + \alpha (t - t_0)] \quad (13)$$

In this equation, P_{STC} is the power of the array under standard test condition, $P_{STC} = 1000 (W / m^2)$, G represents irradiation, t represents ambient temperature, and t_0 stands for standard temperature: $t_0 = 25^\circ C$. Also, α is a factor which depends on panel type. Here, $\alpha = -0.00025 / ^\circ C$ [16]. It should be noted that the original network encounters some voltage issues even in normal operating conditions. By other words, voltage at some buses (e.g., bus4 and bus5 on feeder6) is below the IEEE standard minimum voltage, i.e. 0.95 (p.u.) [19]. Measured voltages at these buses are shown in Fig. 2 for normal operation. It is evident from the figure that voltage in these buses drops significantly, especially during evening time. However, as long as voltage deviation of a customer does not exceed 10%, it would be acceptable as operating voltage [20].

V. SIMULATION RESULTS

In this section, different simulation studies are carried out on the distribution system shown in Fig. 1 to optimally size and locate PV panels. In the first sub-section, simulation results for normal operation, single and multi-objective functions are presented and explained. In the second sub-section, a simulation study is performed to show that power loss minimization is not an effective objective function of a system of this size.

A. Single and Multi-Objective Optimization

Table IV presents the optimal results of each optimization method. It should be mentioned that in this table, the number of 3-kW panels for each bus in the grid is presented. Therefore, not all the households on all buses are capable of installing PV panels. It might be possible for some households to install more than one set of 3-kW panel, while this might not be the case for other households.

TABLE III. NUMBER OF HOUSEHOLDS ON EACH FEEDER.

Feeder	1	2	3	4	5	6
No. of household	20	33	27	28	17	42

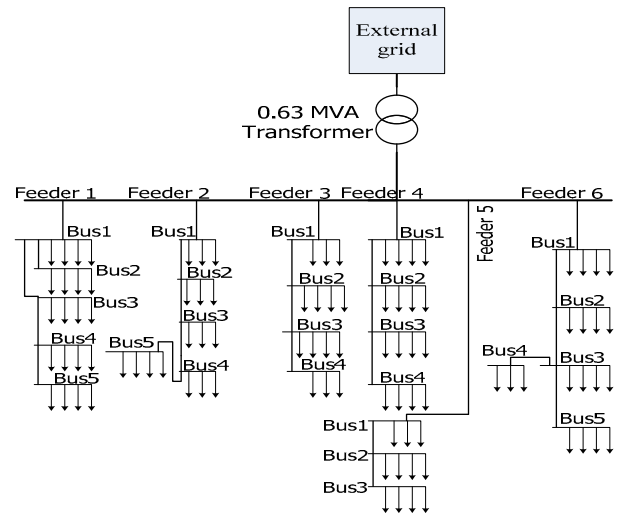


Fig. 1. Grid Configuration.

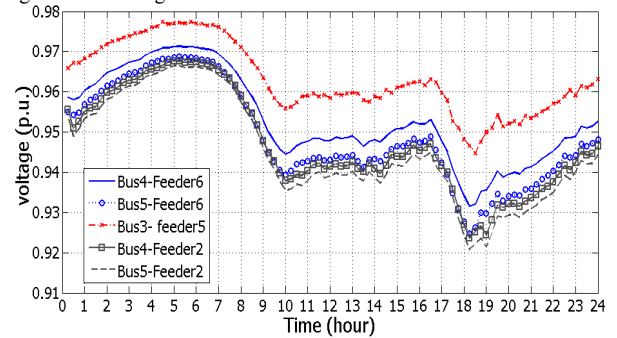


Fig. 2. Voltage of critical buses in normal grid operation.

The number of households on each bus of the grid is also presented in this table. Voltage of grid critical buses for the ‘single-objective’ method is presented in Fig.3. Voltage profiles for the ‘multi-objective’ methods are presented in Fig.4 and Fig.5. From Fig.3, it is obvious that voltage profiles are within standard limits. On the other hand, for ROC method, shown in Fig.4, voltage in two grid buses is more than normal deviation. However, total voltage deviation of these buses is still less than 10% deviation. Therefore, the results are valid, although it’s very marginal. The least change in load profiles might make question these results. On the other hand, voltage profile of RS method, shown in Fig.5, demonstrates that all voltages are within standard limits.

As mentioned earlier, a clear advantage of multi-objective approach to single objective approach is its accurate results. The results of single objective approach would vary in different optimization runs. The change is mainly in the arrangement of the panels, but the number of panels would vary as well. On the other hand, running multi-objective approach for several times has led to similar results. Therefore, the results from multi-objective are more promising and reliable. On the other hand, allocating a higher weight factor to the first objective in ROC method has led to

higher number of PV panels.

On the other hand, as mentioned earlier, the obtained results for ROC approach are marginal values. Therefore, it's a good practice to have the results from both methods. This will help to find out maximum grid capability using ROC method, while achieving a more applicable result by RS method. Also, transformer power profile from the different optimization methods, together with the normal transformer profile, is presented in Fig.6 to illustrate the comparison.

Regarding the optimization results, the grid behavior under high PV penetration can be analyzed. From the voltage profiles, the first point is the positive effect of PV panels on grid voltage profiles. The grid has some voltage issues in normal condition, especially in some critical and heavy loaded buses, presented in Fig. 2, while adding PV panels with any of the proposed methods has improved these voltage profiles. Presence of PV panels, as a power source, covers part of the load demand. As a result, the power flowing from the upper grid decreases. Less power flowing through the lines leads to smaller current in the line cables, which results in less voltage drop.

The effect of the panels on the grid power condition is easy to realize from Fig.6. The normal profile of the grid is also presented in this figure. From the figure, the grid has two peak values during its daily curve; a small peak during morning time, and the main peak which happens in the evening. Adding PV to the grid has changed the profile, shaving the morning peak and lowering the evening peak. Such effect is reasonable, considering the fact that the PV power production depends mainly on the irradiation.

TABLE IV. NUMBER OF 3-KW PV PANELS FOR EACH GRID BUS.

Feeder	Bus	No. of households	Single-objective	ROC approach	RS approach
1	1	2	2	4	4
	2	5	5	10	10
	3	4	4	8	4
	4	6	6	6	9
	5	3	3	0	0
2	1	5	5	10	10
	2	6	6	12	12
	3	9	9	10	13
	4	7	5	3	6
	5	6	6	6	1
3	1	7	5	14	14
	2	7	6	14	14
	3	6	0	10	9
	4	7	4	12	4
4	1	8	4	6	6
	2	8	2	16	15
	3	8	4	11	6
	4	4	4	5	3
5	1	5	5	10	10
	2	6	6	8	8
	3	6	6	5	5
6	1	6	6	12	12
	2	7	7	12	12
	3	12	10	18	15
	4	9	7	6	5
	5	8	2	1	3

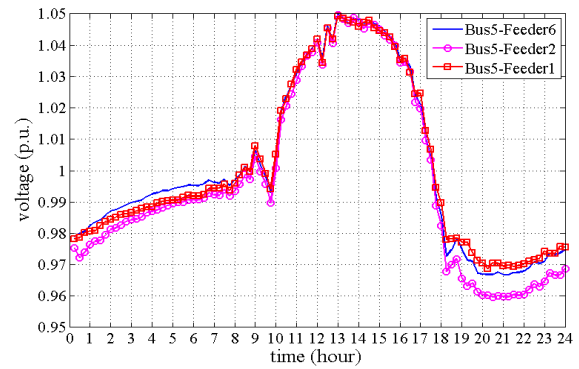


Fig.3. Voltage of buses using single-objective method.

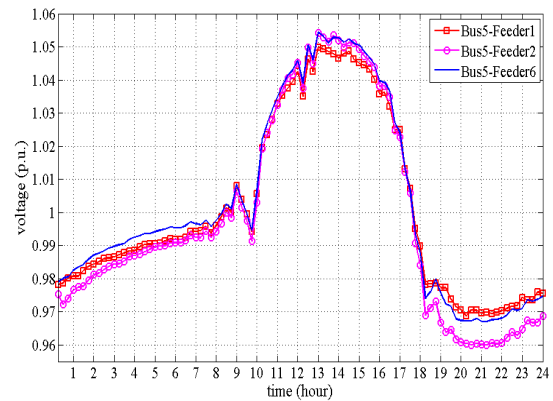


Fig. 4. Voltage of buses using ROC multi-objective method.

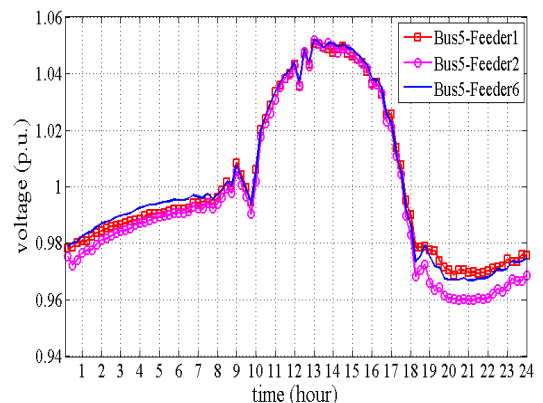


Fig. 5. Voltage of buses using RS multi-objective method.

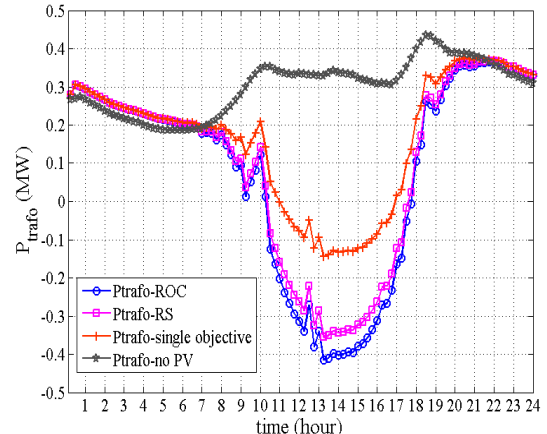


Fig.6. Power profile of the transformer.

As the day comes to its midday time, during which the irradiation is maximum, the PV production increases, while during evening time, due to small irradiation, the PV production is not significant. Of course, the main point in this figure is the negative power flow through transformer. As the PV production increases, it exceeds the grid consumption. Therefore, the extra power of PV panels goes to the upper grid through the transformer. The maximum power to the upper grid is around 105kW for the “single-objective” method, occurring around 1 p.m. to 3 p.m. For the “multi-objective” methods, this value is much more noticeable, with 350kW for RS approach and 410kW for ROC approach. Such significant value of power to the upper grid is a significant potential to be used as a reserve or balancing capacity for the upper grid.

B. Power Loss as Single Objective Function

As mentioned in section II, for multi-objective optimization, the least weight is dedicated to “total power loss”, denoted as $f_3(x)$. The reason is that the total power loss value of the grid is small, as the grid is a small distribution network. To verify that assigning a higher priority to voltage rather than to total loss is reasonable, a separate single objective optimization is done. In this case, “total power loss”, $f_3(x)$, is the objective function. The optimization results are presented in Table V. From the table, it can be realized that the network total power loss has not changed in different optimization repeats, although the arrangement of the panels has changed a bit. Also, to have a more accurate comparison, the total power loss of the network using different optimizations is presented in Table VI. From the results, one can realize the insignificant effect of optimization on grid total loss, comparing to grid normal power loss. The “total power loss” presented in Table V is almost the same as the “total power loss” of other methods in Table VI. Therefore, $f_3(x)$ doesn’t have a significant effect on the optimization results. However, this might not be the case for a large-scale network with significant power loss. So, for different networks with different scales, this fact needs to be addressed.

TABLE V. NUMBER OF 3-KW PV PANELS FOR EACH GRID BUS.

Feeder	Bus	1st optimization attempt	2nd optimization attempt	3rd optimization attempt
1	1	3	1	2
	2	4	6	5
	3	4	3	4
	4	5	6	6
	5	3	3	2
2	1	1	2	2
	2	8	5	7
	3	9	10	9
	4	5	4	5
	5	5	6	5
3	1	7	5	6
	2	2	8	3
	3	7	3	7
	4	4	5	4
4	1	5	4	4

Feeder	Bus	1st optimization attempt	2nd optimization attempt	3rd optimization attempt
	2	5	8	9
	3	5	1	4
	4	3	5	1
5	1	7	6	6
	2	7	8	8
	3	5	5	5
6	1	8	5	6
	2	5	8	7
	3	13	12	13
	4	7	8	7
	5	6	6	6
Total Power loss (kW)		2.4084	2.4082	2.4081

TABLE VI. TOTAL POWER LOSS OF DIFFERENT OPTIMIZATION METHODS.

Method	Single objective	ROC	RS
Total power loss in kW	2.508	2.571	2.493

VI. CONCLUSION

An optimization algorithm is proposed in this paper to find out the maximum number of PV panels in a distribution grid, as well as their arrangement. An optimization algorithm based on GA is proposed. Two approaches were considered to deal with the problem: single-objective approach, and multi-objective approach. Each approach, together with its equations and constraints is explained. The proposed method is applied to a model of a Danish distribution grid to figure out its applicability in dealing with a realistic case. Grid operating requirements are also taken into account. Simulation results verify that the optimization is capable of optimizing PV panel penetration in the grid to maximize reverse power flow and provide ancillary service for the upper grid, without violating grid constraints. Reviewing the results, it can be seen that multi-objective approach presents more promising results comparing to single-objective approach.

Considering the points mentioned in the paper, using storage together with PV panels could help using the panels’ power production in evening peak times. As discussed earlier, maximum power production of panels occur in the middle of the day, while a significant peak of the load profile is during evening time. Therefore, there is a major need for reserve power and balancing power during these hours. Using a storage system enable the grid to deal with such issue. Our next step is to utilize available storages in the grid, such as electric vehicles, to cope with such problem.

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