

Behind-the-Meter Energy Flexibility Modelling for Aggregator Operation with a Focus on Uncertainty

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Abstract—Aggregators are expected to become an inevitable entity in future power system operation, playing a key role in unlocking flexibility at the edge of the grid. One of the main barriers to aggregators entering the market is the lack of appropriate models for the price elasticity of flexible demand, which can properly address time dependent uncertainty as well as non-linear and stochastic behavior of end-users in response to time varying prices. In this paper, we develop a probabilistic price elasticity model utilizing quantile regression and B-splines with penalties. The proposed model is tested using data from residential and industrial customers by assuming automation through energy management systems. Additionally, we show an application of the proposed method in quantifying the number of consumers needed to achieve a certain amount of flexibility through a set of simulation studies.

Index Terms—Flexibility, data-driven modelling, quantile regression, B-splines, industrial and residential consumers

I. INTRODUCTION

To ensure a green transition of the energy system and enable further integration of renewable energy resources into the power grid, new and green flexibility resources will be necessary for the day-to-day grid operation [1]. Advancements in behind-the-meter controllable technologies along with automation, i.e., Energy Management Systems (EMS), enable both traditional electricity consumers and prosumers to provide flexibility. However, the flexibility provided by individual consumers/prosumers is too small for direct participation in the wholesale electricity market. Thus, there is a need for aggregation of these resources.

While there is general agreement that aggregators will become an inevitable part of the future power system, several issues must be addressed before their full potential can be realised in practice. One of the main barriers is the uncertainty associated with Demand Response (DR) of diverse types of consumers, especially through indirect load control by time-varying price signals. Aggregators will need information and methods to evaluate DR from flexible resources to develop

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strategies on how to optimally manage these resources. This calls for probabilistic price elasticity models of prosumers, specifically developed for aggregators.

As of today, several papers have investigated the long-term price elasticity of electricity consumption, e.g., [2] in the US. Unfortunately, the proposed approaches for long-term elasticity cannot be used for day-to-day aggregator operations and are substantially different in nature. In [3], elasticity values and calculation methods are summarised from several studies. The authors found 538 observations of price elasticity estimations in literature, where the average short-term price elasticity was -0.201 . However, the authors did not provide a definition of the short-term price elasticity; thus, the granularity of the price elasticity is unclear. Furthermore, Miller et al. [4] used three data sets for residential consumers in the US to estimate price elasticity, which varies between -0.2 and -0.8 for the given data sets. The variation in price elasticity estimation is an indicator of the uncertainty associated with the underlying phenomenon, in which point estimations may not be able to explain it sufficiently. Hence, there is a need for dynamic and probabilistic elasticity models. Furthermore, a probabilistic consumer flexibility model is essential to apply risk-based methods (e.g., CVaR in [5]) and scenario-based stochastic programming (e.g., [6]) for optimal operation of aggregators. In [7], the DR from households responding to economic incentives for critical load and peak shifting is investigated. The authors concluded that automation strongly increases the price responsiveness, while manual DR can only make long-run adjustments. Thus, automation, e.g., EMS, is necessary to fully realise the potential of demand-side flexibility.

There are also a few papers on short-term price elasticity. J. M. Gillan [8] investigated the short-term price elasticity for residential electricity consumers in California. However, the study only concerns explicit DR. In [9], the authors looked at the price elasticity in the real-time and day-ahead market. These studies consider all customers in the data set with possible effects of customer fatigue in manual DR. Furthermore, in [10], the authors developed an artificial neural networks solution to learn the behavior of a single electricity consumer. The models developed in these studies are deterministic, which

is less useful for the aggregators' operation in an uncertain environment with low profit margin. Probabilistic DR models are presented in the literature, e.g., [11] in which conditional probability density for future demand is used for prediction of demand. However, the model is based on two-way communication and customers react manually to the electricity price signals. A quantile and linear regression-based model of DR is proposed for a single customer in [12] based on the Spanish ADDRESS project, in which 260 residential consumers with Home Energy Management Systems (HEMS) participated. Nevertheless, these studies only considered individual customers from a specific sector, explicit mechanism for DR, and linear modeling approaches.

Furthermore, numerous studies have developed DR models using consumer price elasticity, e.g., [13] in which the short-term price elasticity is assumed to be -0.2 in their probabilistic demand curve model. Also, [14] utilized a function of elasticity to describe DR customer behavior. These studies did not develop a model for the elasticity in particular nor did they discuss the elasticity values/models. Analysing the potential of flexibility to reduce power peak consumption in Northern Europe, [15] found that the flexibility potential varies over different sectors (residential, commercial and industrial). To address the aggregators' interest, the price-load models should therefore be applicable to consumers from different sectors.

This paper aims to provide a methodology for quantifying the consumer/prosumer price elasticity and associated uncertainty. Our research contributes to this field by making hourly price elasticity models using quantile regression and B-splines with penalties. Thereby, we develop a non-linear probabilistic model, reflecting the uncertainty of consumer/prosumer flexibility. We then use the proposed model to quantify the number of consumers required to achieve ± 1 MW flexibility from different sectors in a probabilistic manner. To create a price elasticity model suitable for the aggregators' operation, price-load flexibility data are required. However, due to the scarcity of experimental data involving consumers equipped with automation, synthesized data from [16]–[18] is used for modeling and simulation studies. The contributions of the paper can be summarized as follows:

- We develop hourly flexibility models of consumers that represent non-linearity of the flexibility resources and consumer behavior.
- A probabilistic approach is adopted using quantile regression to model the uncertainty of DR resources, which does not require a probability density function (pdf) as a priori. Therefore, its application is not limited to a certain type of pdf.
- We utilize data from residential and industrial sectors, which are assumed to be equipped with automation through EMS, but still allow for customers' preferences and stochastic behavior.

To the best of our knowledge, such a price-response elasticity model of consumers from different sectors has not been presented in the literature.

The rest of the paper is organized as follows. Section II presents the methodology of applying quantile regression and B-splines with penalties, as well as the methodology to quantify the number of consumers required to obtain a certain amount of flexibility. Section III reports and analyzes the most important findings and Section IV concludes the paper.

II. METHODOLOGY

In this section, we describe the methodology of applying quantile regression (QR) and B-splines with penalties (also known as P-splines) to the price-load data set. Thereafter, we describe the methodology to estimate the number of activated customers required to reach ± 1 MW bid size using the QRs.

In this work, up-regulation refers to a reduction in consumption which is assumed to be a result from positive price on top of the baseline price. Hence, down-regulation means an increase in consumption, which is assumed to be a response to negative incentives. The flexibility behavior appears to be quite different for up- and down-regulation (i.e., to positive and negative price deviations). Therefore, we apply the regression methodology for up- and down-regulation separately.

A. Quantile regression and B-splines with penalties

The magnitude of flexibility, L_h , at a certain hour, h , given a price deviation π , can be described by some unknown pdf, $f(L_h)$ with cumulative density function (cdf) (1).

$$P(L_h|\pi_h) = \int_{-\infty}^{L_h} f(x; \pi_h) dx \quad (1)$$

where the flexibility magnitude, L_h , is the change in load from the baseline at a certain hour, h , while the price, $\pi_h \in [-0.75, 0.75]$ is the deviation in electricity price from a certain baseline electricity price (assumed 2.25 DKK/kWh as in [16]) for the same hour h . For a specific data set C , we can define the conditional distribution as $P(L_h|L_{h,c}, \pi_{h,c})$.

One approach to find the conditional distribution from a data set is to utilize QR. The pdf of DR is unknown and can vary depending on various factors such as type of customers, time of the day and weather. Using QR avoids parametric assumptions on the pdf. Thus, QR is especially well suited for this purpose, since it does not assume any distribution a priori [19].

When applying QR to find the cdf, quantiles may cross. If this occurs, it would imply negative probability according to the definition of quantiles. Hence, a method that does not result in crossings should be applied. One methodology is proposed in [19], in which constraints are applied on the fitted parameters in the linear programming to ensure non-crossing regressions. In this work, we utilize this method through the R package *quantregGrowth* version 0.4.3 [19]–[21]. The QR applied can be described as follows (2).

$$Q_{\tau,h}(\hat{L}_h|L_{c,h}, \pi_{c,h}) = \sum_{j=1}^n \hat{a}_{j,h} B_{j,h}(\pi_h; q) \quad (2)$$

where $Q_\tau(\hat{L}_h|L_{c,h},\pi_{c,h})$ is the estimated function for a quantile τ , given the data set C for hour h . The dimension of the problem is n and is less or equal to the number of price signals. In the loss function for the QR, there is a penalty term $\lambda \sum_j |\Delta_j^d|$, as defined in the R package *quantregGrowth version 0.4.3* [19]–[21], where the order of the difference operator (d) is set to 3. The penalty term penalizes overfitting; thus, it affects the smoothness of the regression. We test for several values of the weight of the penalty, λ , and allow the algorithm to choose the best value through cross validation as described in [19]. We also set the degree of the B-splines to 3. Equation (2) is initially applied to quantiles $\tau = [0.1, 0.2, \dots, 0.9]$

From analyzing the load versus price data set $(L_{c,h}, \pi_{c,h})$, the consumer flexibility appears to be non-linear. Thus, we apply B-splines with penalties using the *ps()* function from the *quantregGrowth version 0.4.3* package in R as in [19].

We apply further modifications in the *ps()* function by limiting the B-splines function to be monotonically non-increasing. This is a reasonable implementation given that the higher the price the lower consumption is expected. If the customers are equipped with EMS, it is not expected that a higher price will give a higher consumption. It should be mentioned that, due to the stochastic behavior of flexible electricity consumers, it can happen that a higher price yields a higher consumption although a lower consumption is expected. However, this would not be driven by the price, but rather happen due to other reasons, e.g., rebound effect of critical loads. Thus, it should not be reflected in the fitted spline functions, but rather by the uncertainty, i.e., the shape of the cdf. Alternatively, applying more explanatory variables, such as the rebound effect, may fix this issue, although this has not been investigated in this article.

B. Estimation of activated customers

As most markets require a minimum bid size to participate in the market, aggregators are interested in estimating the required number of customers to reach a minimum bid size. This estimation can be utilized to determine the number of customers in the pool. Alternatively, it can be used to evaluate the participation of customers from different categories or sectors by estimating the number of customers that should be activated at a certain hour from an already existing customer pool. Here, we set the required bid size of a hypothetical market to ± 1 MW and describe how to make such estimates from the proposed QR models.

As described above, the cdf for a certain price can be extracted from the QR. Here, we demonstrate this by extracting the cdf for 0.5 DKK/kWh and -0.5 DKK/kWh price variations, which are the medians for the positive and negative price deviations. The extracted values are samples from the cdf for their respective τ . To get a full cdf, we make a piecewise linear regression between the extracted points. Since the uncertainty is higher in the tail probabilities of the cdf, where the cdf tends to vary more, we add quantiles

$\tau_k = [0.01, 0.05, 0.95, 0.99]$ to the QR model. To deal with the end points of the QRs, i.e., from $\tau = 0.01$ to 0 and $\tau = 0.99$ to 1, a different approach is needed. Since the data in this case behave well and do not reach negative loads for negative prices, it is possible to make a linear regression between the QR for $\tau = 0.01$ and zero for down regulation. The same is valid for up-regulation and $\tau = 0.99$, since the load at $\tau = 1$ does not go above zero. For the case of down-regulation and $\tau > 0.99$, there is no natural maximum that can be extracted from the data. Therefore, we fit an exponential function to describe the cdf from $\tau = 0.99$ to $\tau = 1$. For the up-regulation, it is assumed that the total consumption will not be negative and a linear regression is made from $Q_{\tau=0.01}(L_h)$ to the negative value of what they are already consuming at that hour, i.e., the base load, $-L_{base,h}$.

Using a uniform distribution, $\mathcal{U}(0, 1)$, we then simulate from the cdf until 1 MW or -1 MW is reached. In other words, we find i such that (3) for down-regulation is satisfied for $L_{bid} = 1MW$; similarly for up-regulation that (4) is satisfied for $L_{bid} = -1MW$.

$$L_{bid} \geq \sum_{j=1}^i F^{-1}(r_j) \quad (3)$$

$$L_{bid} \leq \sum_{j=1}^I F^{-1}(r_j) \quad (4)$$

where $r_j \sim \mathcal{U}(0, 1)$. This is repeated 1000 times, giving an estimate of mean and variance for how many customers are required to reach 1 or -1 MW, respectively, for a certain customer cluster and certain hour, given a price deviation.

III. RESULTS AND ANALYSIS

In this section, we present some of the findings from applying the methodology described in Section II. In addition, we present the results of the simulations of activated customers from the presented cases.

In the data utilized for this study, the baseline electricity price is assumed to be 2.25 DKK/kWh with a variable price component, $\pi_h \in [-0.75, 0.75]$. Consumers categories are aggregated to form 3 clusters (residential, light industry, heavy industry). Further details of the data are described in [22], accompanied by [16]–[18].

A. Flexibility model of demand with QR

In this work, we apply 9 QRs such that τ_k takes on values from 0.1 to 0.9, linearly spaced with 0.1 interval. Of course, when applying the methodology to a specific case, this could be changed depending on the aggregator's needs in terms of uncertainty analysis.

In Fig. 1, the results for the residential cluster at hour 12 are provided, while the results for the heavy industry cluster can be seen in Fig. 2. In both figures, it is shown that a point estimation would not be sufficient and that a confidence interval would not give the full picture of the possible outcomes of flexibility. Meanwhile, the QR curves span over the

entire data set. It can also be seen that the response function is not linear, which supports the necessity of a non-linear method. Additionally, it can be observed that crossings of the QR curves are avoided through the application of the `quantregGrowth` package [19], [21]. From both graphs (Figs. 1 and 2), cdfs can be extracted by simply fixing the electricity price deviation and obtaining the flexibility magnitude from the QRs, arranging the values of τ_k in an increasing order.

For cluster 1 (Fig. 1), the variance is quite high for up-regulation compared to cluster 3. For the case of down-regulation, the flexibility has more dense QR curves in the upper region of the data set. This implies a more skewed distribution function, with higher probabilities of the customers actively responding. As a result, the widely-used normal distribution in such studies does not properly describe the flexibility. This justifies the application of QRs with which a priori pdf is not required.

Looking at cluster 3 (Fig. 2), however, it can be noticed that the variance is higher for the down-regulation. It is also seen that the pdf is skewed towards non-responsive behavior, unless the price deviation is high. For the up-regulation, on the other hand, the observed reactions are smaller in magnitude but more certain. Here, the response also appears to have a bimodal distribution that is well captured by the QR curves by being more dense around these lines. If simulations were made from the up-regulation case, two most probable scenarios would be observed.

One reason for this bimodal distribution for up-regulation in cluster 3 could be that two categories in the cluster, “Non-metallic” and “Other industries”, have quite similar behavior, whereas the behavior of the category “Chemical” differs significantly from the other two. This can be seen in the assumptions made for the categories (Table I in [16]).

Furthermore, the lack of flexibility to down-regulation prices can be explained by the industrial customers’ strict technical and operational constraints compared to the residential ones.

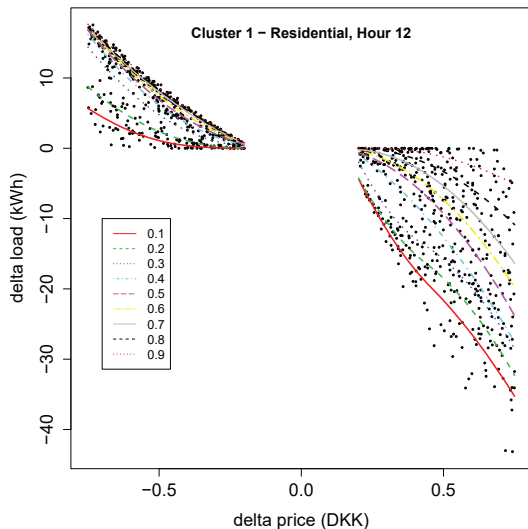


Fig. 1. Flexibility of 280 residential customers in cluster 1. The graph shows hour 12 with 9 quantile regressions.

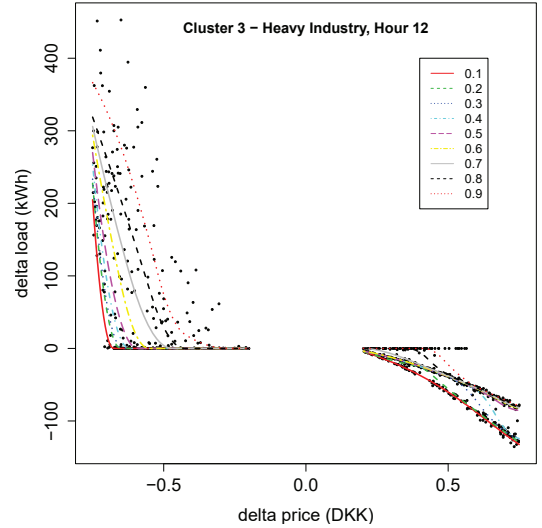


Fig. 2. Flexibility of 210 heavy industry customers in cluster 3. The graph shows hour 12 with 9 quantile regressions.

However, they are business-driven: for the right price deviation at the right time, they react to optimize their energy consumption expenditure. In Fig. 2, it can be seen that for hour 12, it is not favorable for the industrial consumers to increase their consumption, unless the price deviation is very high.

The fact that industries have technical constraints and generally operate in more safety-driven ways than residential consumers can also be seen in the comparison between clusters 1 and 3. Thus, the response from the residential cluster has a higher variance, while for the heavy industry cluster, the patterns are more recognizable, probably due to their routine day-to-day operation.

B. Customer activation

In this section, we present the results from the 1000 simulations for up- and down-regulation respectively, to achieve ± 1 MW in load deviation through the flexibility models, as described in Section II. To better account for the uncertainty in the lower and upper end of the cdf, we also include QRs for $\tau_k = [0.01, 0.05, 0.95, 0.99]$, as described in Section II.

The results of the simulations for cluster 1 (residential), are presented in Fig. 3. In general, it can be seen that the uncertainty in the number of required customers is larger for up-regulation compared to down-regulation. This is a direct result from the larger variance on the up-regulation side in Fig. 1. It can also be observed that significantly more customers are needed for down-regulation compared to up-regulation for hours 2, 3, and 4, as well as hours 12, 13, and 14. This could be due to the rebound effect and the fact that the residential customers are asleep in the earlier hours or not at home in the middle of the day. Thus, there is no need for increasing their electricity consumption.

For cluster 2 (light industry), the results are visualized in the box-plot of Fig. 4, where it can be observed that significantly fewer customers are required to obtain -1 MW than 1 MW of flexibility. The number of electricity customers required

for down-regulation in hours 1, 8, 17, and 24 is high, which means that there is not much flexibility (or willingness by the consumers to provide flexibility) to be activated; thus lower commitment for the aggregator in the wholesale market can be suggested. For hours 6, 10, 11, 13, 20, 21, 22, and 23, we stopped the simulations at 2.8 million customers without reaching 1 MW. It should, however, be noted that for a larger price deviation, the 1 MW flexibility could be achieved.

For cluster 3 (heavy industry) in Fig. 5, it can be seen that fewer customers are required to achieve -1 MW than 1 MW. This is in line with what is shown in Fig. 2. For a price deviation of -0.5 DKK/kWh, there is a 70-80% probability that there will be no reaction or a very small reaction from the customer cluster. On the other hand, for 0.5 DKK/kWh, the probability that the cluster is not responsive to the incentives is less than 10%. It can also be seen that the number of required customers for both up- and down-regulation decreases over the day. The base-load consumption is lower during the early hours of the day and higher in the later hours. Seeing higher activity in the later part of the day suggests that there are more active loads to be offered for both up- and down-regulation.

Overall, these results could be further used by the aggregators to target the right consumers for flexibility provision. Aggregators can get an insight into how large a customer pool from different clusters they need and their associated uncertainty. In other words, they can estimate the number of activated customers that are required at each hour in the best and worst case scenarios. Such results can be used in operational risk assessments and meeting the minimum bid requirements. As another advantage of the proposed approach, the aggregators receive valuable insights into cluster integration. For instance, while more customers in cluster 2 are required for up-regulation in hours 19, 20, and 21, consumers in cluster 3 can provide a lot more flexibility in the same period of time. These complementary effects can be exploited by the aggregators.

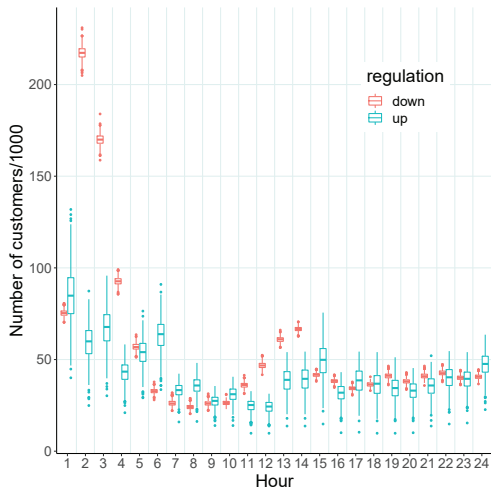


Fig. 3. Number of activated electricity customers required to achieve ± 1 MW in DR from cluster 1 (residential cluster).

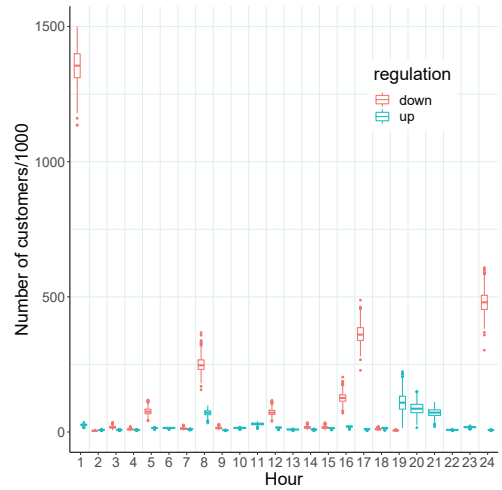


Fig. 4. Number of activated electricity customers required to achieve ± 1 MW in DR from cluster 2 (light industry cluster).

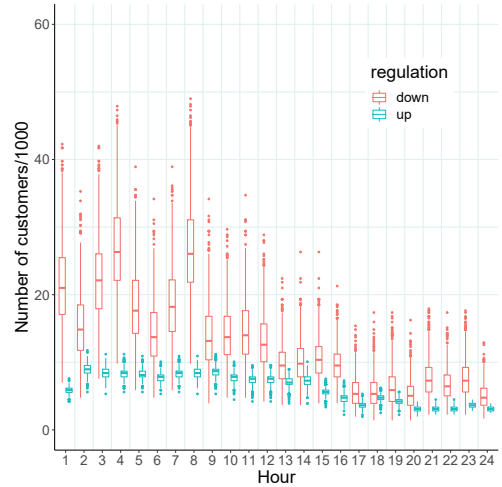


Fig. 5. Number of activated electricity customers required to achieve ± 1 MW in DR from cluster 3 (heavy industry cluster).

IV. CONCLUSION

This paper presents an hourly price load model for implicit flexibility provision. QR and B-splines with penalties were applied to achieve a non-linear probabilistic model to capture the variance and uncertainty in the data. A future scenario was assumed, in which the customers are equipped with EMS and the model was applied to both residential and industrial electricity customers. From the QRs, the cdf for ± 0.5 DKK/kWh was extracted. From this study, we observed a higher uncertainty for up-regulation compared to down-regulation from the residential cluster. We also discovered that fewer customers were required for up-regulation compared to down-regulation from both light and heavy industry clusters.

From an aggregator's perspective, the simulation results can be further employed for risk assessments. For instance, scenarios can be defined, such as best, worst and most probable scenarios of required activated customers from an already existing customer pool. Additionally, the results may assist

aggregators in determining the customer segments with the highest flexibility and willingness for their business.

Furthermore, QR and extracted cdfs could be used for scenario generation for flexibility. Alternatively, they can also be utilized to generate inputs for CVaR models.

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