Analysis of rebound effect modelling for flexible electrical consumers ⋆

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Abstract: Demand response (DR) will be an inevitable part of the future power system operation to compensate for stochastic variations of the ever-increasing renewable generation. A solution to achieve DR is to broadcast dynamic prices to customers at the edge of the grid. However, appropriate models are needed to estimate the potential flexibility of different types of consumers for day-ahead and real-time ancillary services provision, while accounting for the rebound effect (RE). In this study, two RE models are presented and compared to investigate the behaviour of flexible electrical consumers and quantify the aggregate flexibility provided. The stochastic nature of consumers' price response is also considered in this study using chance-constrained (CC) programming.

Keywords: Rebound effect; Flexible consumers; Mixed-integer linear program; Chanced-constrained programming; Demand response; dynamic prices.

NOMENCLATURE

Sets:

- T Set of time periods, indexed by t, $t \in [1, \dots, \tau]$.
- J Set of consumers types, indexed by j.
- α Types of regulation, i.e., up- or down-regulation.
- D Set of days, indexed by t_D, $t_D \in [1, \dots, \frac{\tau}{24}]$.

Parameters:

- R_j Maximum rebound effect duration for consumer type j [h].
- $\boldsymbol{\lambda}^{\text{base}}$ Base-line electricity price [DKK cent/Wh].
- $\boldsymbol{\Delta\lambda}_t^{\text{u}}, \boldsymbol{\Delta\lambda}_t^{\text{d}}$ Dynamic electricity price for up- and down-regulation at time t [DKK cent/Wh].
- $\underline{\boldsymbol{\Delta\lambda}}_j^{\alpha}, \overline{\boldsymbol{\Delta\lambda}}_j^{\alpha}$ Minimum and maximum delta prices for regulation type α of consumer type j [DKK cent].
- $\mathbf{L}_{t,j}^{\text{base}}$ Base-line consumption of consumer type j at time t [W].
- $\mathbf{L}_{t,j}^{\min}, \mathbf{L}_{t,j}^{\max}$ Minimum and maximum electricity consumption of consumer type j at time t [W].
- $\mathbf{a}_{t,j}^{\alpha}, \overline{\mathbf{a}}_{t,j}^{\alpha}$ Willingness and maximum willingness of consumer type j to provide flexibility type α at time t [p.u.].
- $\mathbf{r}_j^{\alpha}$ Ramp-rate of consumer type j for regulation type α [W/h].
- $\mathbf{n}_j^{\alpha}$ Maximum number of activations for consumer type j to provide flexibility type α.
- $\underline{\mathbf{d}}_j^{\alpha}, \overline{\mathbf{d}}_j^{\alpha}$ Minimum and maximum continuous flexibility duration of consumer type j when activated to provide flexibility type α [h].

Variables:

- $L_{t,j}^{\alpha}$ Flexibility of end-users' category j at time t for regulation type α [W].
- $w_{t,j}, v_{t,j}, x_{t,j}$ Binary variables defining the regions of the overall flexibility provided up to time t by end-users' category j.
- $u_{t,j}^{\alpha}$ Binary variables, indicating flexibility status of end-users' category j at time t for regulation type α.
- $y_{t,j}^{\alpha}, z_{t,j}^{\alpha}$ Starting and stopping binary variables of end-users' category j at time t for flexibility type α.
- $\Theta_{t,j}$ Overall flexibility provided at time t from consumer type j [W].

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1. INTRODUCTION

Demand response (DR) programs are solutions that target changes in the power consumption of electrical consumers through economic incentives. With the higher penetration of renewable energy resources in the system, such programs are becoming a popular solution to better meet the stochastic electricity generation and support the power system operation. Several DR solutions have been proposed in literature, e.g., by offering long-term contracts, or by broadcasting dynamic prices.

In the long term contracts, consumers allow an external operator to decide about their electricity consumption in exchange for an economic incentive (see e.g. Bitar and Low (2012)). In dynamic price schemes, however, consumers receive a time-varying price by their home-energy management system (HEMSs) and decide individually their electricity consumption schedule to minimise overall cost while preserving comfort and privacy (see e.g. Gillan (2017)). As the latter does not restrict end-users' autonomy or independence, dynamic price schemes are most likely to be accepted by consumers. As a result, we focus on DR programs based on the dynamic prices as the control signal in this study.

In order to fully exploit the potential of DR programs, it is important for the operators (i.e., DR aggregators and system operators) to understand how consumers respond to prices on an aggregated level. Such an understanding can facilitate the formulation of proper dynamic prices that achieve a certain change in consumption from consumers. Furthermore, it can support the operators in quantifying the potential flexibility that can be achieved from DR programs and better allocate the reserve requirements for the power system operation.

Of particular importance in this matter is the rebound effect (RE), which consists of the change in consumers' consumption due to previous and future price reactions and is related to the technical constraints of loads and consumers' preferences. RE represents the power consumption increase (decrease) that follows an event of up- (down-) regulation, for which the consumption is decreased (increased) (O'Connell et al. (2014)). In the literature, RE is mainly investigated in relation to thermal loads or refrigerators (O'Connell et al. (2014)) that will need to recover their consumption immediately after a decrease in their consumption by their own dynamics. In this paper, we extend this concept to shiftable loads (i.e., washing machines, as discussed in Klaassen et al. (2016)) as they follow a similar behaviour. Both thermal and shiftable loads can be modelled by consumers that reduce (increase) their baseline consumption scheduled at a certain time and consume more (less) in the following time steps. The main difference between the types of loads is the time period for which the RE phenomenon must be completed (i.e., refrigerators have faster dynamics than washing machines). Therefore, we can formulate a general mathematical model of the RE for both thermal and shiftable loads, where the different dynamics impact appears in the maximum RE duration parameter. In this paper, the RE is formulated assuming that the increase and decrease in consumption perfectly compensate each other in a certain period of time, defined as perfect RE. Although such an assumption might not be realistic for all types of loads, a practical model of such requires detail models and field data. An alternative representation to perfect RE will be investigated in our future studies.

Despite the importance of quantifying consumers' price response, proper RE modelling has scarcely been investigated and the majority of studies evaluated RE in relation to the change in energy efficiency (Greening et al. (2000)).

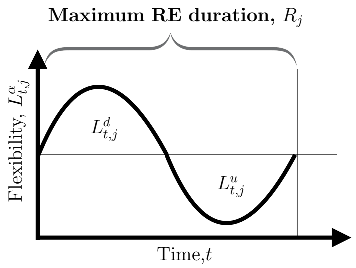

Fig. 1. Basic concept of perfect RE.

In Georges et al. (2017), RE was modelled for a pool of residential heat-pumps, assuming that an operator could decide the consumption of a pool of consumers. In that study, the RE was modelled by a delay period with no deviations from the baseline consumption, and a payback period during which deviations in consumption occurred to allow the heat-pumps to return to their baselines. Although the study evaluated the dynamics of loads for a pool of residential heat-pumps, additional studies are needed to quantify the aggregate RE impact of different types of consumers.

The main contributions of this paper can be summarised as follows. First, we present two different formulations to model RE on an aggregated level with different types of consumers using mixed-integer linear programming (MILP). Second, we compare both formulations and use them to quantify the overall flexibility that can be achieved from a heterogeneous pool of consumers. Furthermore, we benchmark the two formulations with each other in terms of computational time and model sizes.

The paper is organised as follows: in Section II, the two formulations of RE are explained; in Section III, results of the models are presented and discussed; in Section IV, we summarise the conclusions.

2. MODELLING

We start by briefly explaining the concept of perfect RE. In Fig. 1, the condition of perfect RE is shown for a consumer of type j that provides regulation in α direction at time t. Load flexibility $L^{\alpha}_{t,j}$ can be provided either for up-regulation ($\alpha = u$) or down-regulation ($\alpha = d$).

In Fig. 1, the increase in electricity consumption achieved from consumers responding to a DR program (i.e., $L^{d}_{t,j}$) is always compensated with a decrease (i.e., $L^{u}_{t,j}$) of the same magnitude in the following time steps. This concept is also valid vice versa, where an increase in electricity consumption follows a previous decrease. The duration period for which the RE must be completed depends on the characteristics of the loads and is here defined as maximum of R_j periods for each consumer type j. If we define $\Theta_{t,j} = L^{d}_{t,j} - L^{u}_{t,j}$, the general RE condition can be formulated as $\sum_{t}^{t+R_j} \Theta_{t,j} = 0$. Depending on the type and the dynamics of a load, it is possible to consider the RE duration as static (i.e., for specific time periods) or dynamic (i.e., allowing a more adaptable scheduling of the flexibility). These two different formulations are presented in the Sections 2.1 and 2.2, while the overall aggregation model of the consumers is given in Section 2.3.

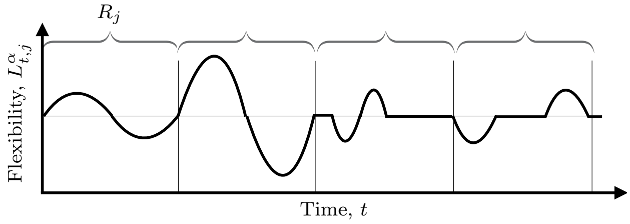

Fig. 2. Rebound effect for specific time steps.

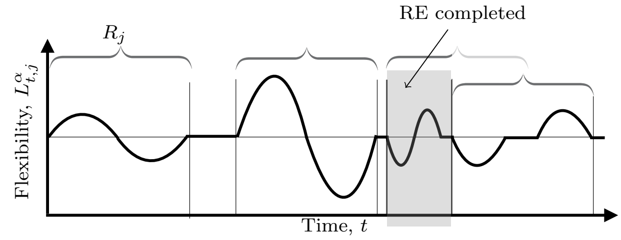

Fig. 3. Rebound effect for duration.

2.1 Modelling rebound for static RE duration

In this subsection, we model the RE for consumers that require a static RE duration. An example of this condition could be the charging of an electric vehicle (EV) that starts at 11:00 and needs to be completed by 16:00. In Fig. 2, this RE model is graphically presented. With static time steps, the RE can be formulated as:

$$\sum_{t'=(t-1)R_j+1}^{(t-1)R_j+R_j} \Theta_{t,j} = 0 \qquad (1a)$$
$$\forall t : [t \in T, (t \cdot R_j \leq \tau)], j$$

For this formulation, we divide the time set T by the RE duration of each type of consumers j. In this manner, we set the time intervals for which the total amount of flexibility provided by consumers type j up to time t must be nullified. Therefore, in Eq. 1a, the overall flexibility provided by each type of consumers j must be nullified within each RE cycle.

2.2 Modelling rebound for dynamic RE duration

Not all loads can be represented by a static RE model. An example is thermal loads, which can always provide flexibility as long as some operational constraints are respected. For this type of loads, the condition that the perfect RE is completed must be imposed only when flexibility is being provided. This concept is visualised in Fig.3.

In Fig. 3, the RE duration is set dynamically whenever regulation is provided. However, the RE must be completed at least once within R_j (i.e., to guarantee a certain temperature in the room). When the perfect RE is achieved (highlighted as light grey area in Fig. 3), a new RE cycle can be started. This RE model can be formulated as:

$$\epsilon x_{t,j} - M w_{t,j} - \epsilon v_{t,j} \leq \sum_{t'=1}^{t} \Theta_{t',j} \qquad \forall t, j \qquad (2a)$$

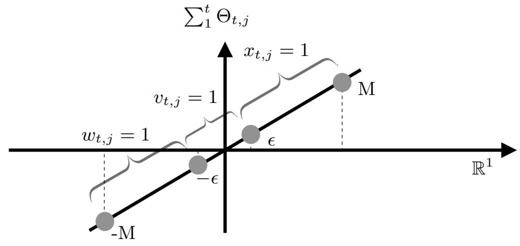

Fig. 4. Definitions of three possible regions of $\sum_1^t \Theta_{t,j}$.

$$\sum_{t'=1}^{t} \Theta_{t',j} \leq -\epsilon w_{t,j} + M x_{t,j} + \epsilon v_{t,j} \qquad \forall t, j \qquad (2b)$$

$$x_{t,j} + w_{t,j} + v_{t,j} = 1 \qquad \forall t, j \qquad (2c)$$

$$v_{t-1,j} - v_{t,j} \leq \sum_{t'=t}^{t+R_j} v_{t',j} \quad ,\forall t : [t \in T, t \leq \tau - R_j], j \qquad (2d)$$

The total amount of flexibility provided by consumer type j until time t, $\sum_1^t \Theta_{t,j}$, can either be zero (when the amounts of down- and up-regulation perfectly compensate each other), positive or negative. For this reason, we define three possible regions for the value of $\sum_1^t \Theta_{t,j}$ in Eqs. (2a)-(2b). These regions are modelled through three binary variables where only one of them can be non-zero at time t. $x_{t,j}$=1 represents the region where $\sum_1^t \Theta_{t,j}$ has positive values; $w_{t,j}$=1 describes the region where $\sum_1^t \Theta_{t,j}$ has negative values and $v_{t,j}$=1 models the region where $\sum_1^t \Theta_{t,j}$ is zero (see Fig. 4). To define the three possible regions in the model, we use a big-M formulation, where M is a large constant and ϵ is small constant. Eq. (2c) guarantees that $\sum_1^t \Theta_{t,j}$ can only be in one of these regions at time t. Eq. (2d) ensures that when consumers start providing flexibility, the RE must be perfectly completed at least once within R_j periods.

2.3 Quantifying the flexibility provision

In this subsection, we provide the overall MILP that can schedule the flexibility provision to achieve cost minimisation for each customer type j (see De Zotti et al. (2018)).

$$\min_{L^{\alpha}_{t,j}} \quad \sum_{t=1}^{\tau} \big(\boldsymbol{\lambda}^{\text{base}} + \boldsymbol{\Delta\lambda}^{u}_{t} + \boldsymbol{\Delta\lambda}^{d}_{t}\big) \sum_{j=1}^{J} \big(\boldsymbol{L}^{\text{base}}_{t,j} + \Theta_{t,j}\big) \qquad (3a)$$

$$\text{s.t.} \quad -\mathbf{r}^{\alpha}_{j} \leq L^{\alpha}_{t+1,j} - L^{\alpha}_{t,j} \leq \mathbf{r}^{\alpha}_{j} \qquad \forall t, j, \alpha \qquad (3b)$$

$$\big(\mathbf{L}^{\max}_{t,j} - \mathbf{L}^{\text{base}}_{t,j}\big) = \Theta^{d} \qquad (3c)$$

$$\big(\mathbf{L}^{\text{base}}_{t,j} - \mathbf{L}^{\min}_{t,j}\big) = \Theta^{u} \qquad (3d)$$

$$0 \leq L^{\alpha}_{t,j} \leq u^{\alpha}_{t,j} \Theta^{\alpha} \mathbf{a}^{\alpha}_{t,j} \qquad \forall t, j, \alpha \qquad (3e)$$

$$u^{d}_{t,j} + u^{u}_{t,j} \leq 1 \qquad \forall t, j \qquad (3f)$$

$$y^{\alpha}_{t,j} - z^{\alpha}_{t,j} = u^{\alpha}_{t,j} - u^{\alpha}_{t-1,j} \qquad \forall t, j, \alpha \qquad (3g)$$

$$y^{\alpha}_{t,j} + z^{\alpha}_{t,j} \leq 1 \qquad \forall t, j, \alpha \qquad (3h)$$

$$\sum_{t'=(t_D-1)24+1}^{(t_D-1)24+24} y^{\alpha}_{t',j} \leq \mathbf{n}^{\alpha}_{j} \qquad \forall j, \alpha, t_D \qquad (3i)$$

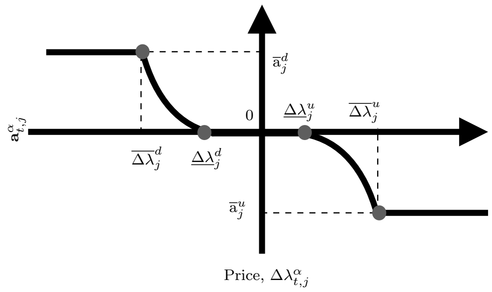

Fig. 5. Modelling consumers' willingness, $\mathbf{a}^{\alpha}_{t,j}$, as a function of delta price.

$$\sum_{t'=t}^{t+\underline{\mathbf{d}}^{\alpha}_{j}} u^{\alpha}_{t',j} \geq \underline{\mathbf{d}}^{\alpha}_{j} y^{\alpha}_{t,j} \tag{3j}$$

$$\forall t : [t \in T, (t + \overline{\mathbf{d}}^{\alpha}_{j} < \tau)], j, \alpha$$

$$\sum_{t'=t}^{t+\overline{\mathbf{d}}^{\alpha}_{j}} z^{\alpha}_{t',j} \geq y^{\alpha}_{t,j} \tag{3k}$$

$$\forall t : [t \in T, (t + \underline{\mathbf{d}}^{\alpha}_{j} < \tau)], j, \alpha$$

$$\sum_{t=1}^{\tau} \Theta_{t,j} = 0, \qquad \forall j \tag{3l}$$

The objective function (3a) minimises the cost of customer type j for purchasing electricity within the planning horizon of τ periods. In the objective function, the electricity price consists of a base-line component λ^{base} (that covers fixed costs and taxes) and a dynamic component, which might be positive ($\Delta\lambda^{\text{u}}_{t}$) or negative ($\Delta\lambda^{\text{d}}_{t}$) depending on the type of regulation needed. The dynamic price components are assumed to achieve a certain change in consumption from the consumers. The constraints are formulated as follows: Eq. (3b) is related to the up- and down-ramp limits of the flexible loads, which are represented for each consumer type j by the ramp-rate parameter $\mathbf{r}^{\alpha}_{j}$; Eq. (3c)-(3e) enforce lower and upper bounds on the amount of flexibility that can be provided by each consumer type j. Note that the minimum and maximum load for each consumer type j at time t, i.e., $\mathbf{L}^{\min}_{t,j}$ and $\mathbf{L}^{\max}_{t,j}$, represent the lowest and highest possible consumption that each consumer type can sustain at time t. In other words, they define the demand flexibility that can be achieved from each consumer type in a specific time.

In Eq. (3e), $\mathbf{a}^{\alpha}_{t,j}$ represents the willingness of consumers to provide DR for regulation type α. It is a function of the price and can vary between -1.0 and 1.0. Beyond a certain price threshold, which we define as $\underline{\Delta\lambda}_{j}$, consumers have a willingness of:

$$\mathrm{a}^{\alpha}_{t,j} = \overline{\mathrm{a}}^{\alpha}_{j} \Big(\frac{\Delta\lambda^{\alpha}_{t} - \underline{\Delta\lambda}^{\alpha}_{j}}{\overline{\Delta\lambda}^{\alpha}_{j} - \underline{\Delta\lambda}^{\alpha}_{j}} \Big)^{\gamma} \tag{4}$$

However, beyond a certain cap price, denoted by $\overline{\Delta\lambda}_{j}$, price response saturates and no additional flexibility can be provided. The parameter $\mathbf{a}^{\alpha}_{t,j}$ is also illustrated in Fig. 5.

In order to include the stochastic behaviour of consumers, we apply chance-constrained (CC) programming to Eq. (3e) for a confidence level of $\beta = 95\%$. In order to do that, we assume that $\mathrm{a}^{\alpha}_{t,j}$ follows a normal distribution, as it is related to human behaviour. Eq. (3e) is therefore reformulated to:

$$0 \leq L^{\alpha}_{t,j} \leq \mu^{\alpha}_{\mathbf{a}} u^{\alpha}_{t,j} \Theta^{\alpha} + \sigma^{\alpha}_{\mathbf{a}} u^{\alpha}_{t,j} \Theta^{\alpha} \Phi^{-1}_{\boldsymbol{\beta}} \tag{5}$$

In this formulation, $\mu^{\alpha}_{\mathrm{a}}$ and $\sigma^{\alpha}_{\mathrm{a}}$ represent the mean value and the standard deviation of $\mathrm{a}^{\alpha}_{t,j}$. For more information about the use of CC programming in this setting, the modelling of $\mathbf{a}^{\alpha}_{t,j}$ and the validity of the normality assumption, please refer to De Zotti et al. (2018).

Eq. (3f) ensures that only one type of flexibility (i.e., up- or down-regulation) is provided by consumer type j at time t; Eq. (3g) represents the flexibility activation and deactivation for consumer type j at time t; Eq. (3h) implies that flexibility provision cannot be activated and deactivated at time t for consumer type j; Eq. (3i) enforces a limit on the number of times that a certain consumer type can be activated in a day. Eq. (3j)-(3k) refer to the minimum and maximum duration for which the load response can be sustained. Eq. (3m) guarantees that the overall flexibility provided is nullified over the time period.

For the numerical results, we combine the overall model with the two types of RE modelling. In the remainder of this paper, the model with static RE duration is referred to as model A and it consists of Eq. (1a) and Eq. (3a-3l). The model with dynamic RE duration is referred to as model B and involves Eq. (2a-2d) and Eq. (3a-3l).

3. NUMERICAL RESULTS

In this section, we provide the numerical results to quantify the overall flexibility provision when considering different RE models and a computational study of the models. To solve the MILP problem, we use actual Danish electricity consumption data for 29 different consumers' categories (i.e., residential, commercial and industrial), as discussed in De Zotti et al. (2018). The data have been collected by Energinet and Dansk Energi during the Elforbrugspanel project and are available at Elforbrugspanel (2018). The values of the parameters which have been used in the simulation studies can be found in De Zotti et al. (2018).

In this study, we investigate the two models A and B for 2 days (i.e., $\tau = 48$ hours) for different delta price sets and temperature settings to identify the range of flexibility that can be provided at each hour. Therefore, we generate 1000 random delta price sets with uniform distribution, assuming that λ^{base} is equal to 2.25 DKK/kWh and that the dynamic price set component varies within $\underline{\Delta\lambda}^{\alpha}_{j} = 0.2$ DKK/kWh and $\overline{\Delta\lambda}^{\alpha}_{j} = 0.75$ DKK/kWh.

Although $\mathrm{a}^{\alpha}_{t,j}$ is represented only as function of the price in Eq. (4), it is possible to extend its modelling and include the effect of the outdoor temperature. In fact, the temperature can have a close relationship to the electricity consumption (see Beccali et al. (2008)). For example, in summer, extreme temperatures require higher electricity consumption for cooling, as shown in Fig. 6. Therefore, there might be higher chances for the operator that consumers are willing to provide DR under the

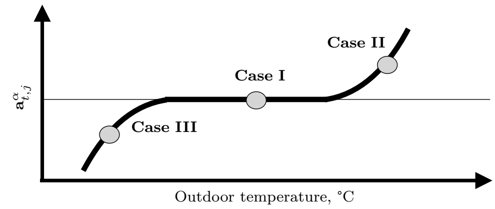

Fig. 6. Relationship between temperature and willingness parameter $\mathbf{a}^{\alpha}_{t,j}$.

Table 1. Average values of the flexibility provided and of the overall electricity cost.

RE model	Case	Flexibility provided [MWh]	Electricity cost [million DKK]
A	I	600.31	380.47
B	I	874.59	380.11
A	II	714.67	380.33
B	II	1032.10	379.92
A	III	482.89	380.60
B	III	714.50	380.30

condition that their comfort is guaranteed. To include the outdoor temperature in the $\mathrm{a}^{\alpha}_{t,j}$ formulation, we multiply $\overline{\mathrm{a}}^{\alpha}_{j}$ of Eq. 4 by a correcting parameter, ν. We consider three cases of outdoor temperatures: Case I deals with a base-line temperature and refers to the initial mean value of $\overline{\mathrm{a}}^{\alpha}_{j}$ (i.e., ν=1); Case II considers $\overline{\mathrm{a}}^{\alpha}_{j}$ for higher outdoor temperature (i.e., ν=1.1); Case III models $\overline{\mathrm{a}}^{\alpha}_{j}$ for lower outdoor temperature (i.e., ν=0.9).

We modelled both MILPs in GAMS 24.9.1 using Gurobi 8.1.0 as solver. The experiments where carried out on Intel(R) Core(TM) i7-2600 CPU 3.40GHz processor with 16 GB of RAM.

3.1 Socioeconomic analysis

In this subsection, we investigate the overall benefits achieved by the operator (by procuring flexibility) and the consumers (by minimising their electricity cost) through the proposed DR program with different RE models. The overall electricity cost for the consumers and the aggregate amount of flexibility achieved are given in Table 1. From the table, it can be seen that model A achieves a lower amount of flexibility than model B (about 31.3%). This is due to the fact that consumers in model A are more constrained by the RE formulation. Consequently, consumers in model A pay a higher cost for procuring electricity (i.e., 0.1%, 360,000 DKK). However, the difference in electricity cost depends on the price formulation (which in this case are capped and, therefore, limiting the cost reduction). From Table 1, it can be further seen that the temperature affects the overall flexibility and cost, where Case II leads to an amount of flexibility that is around +45% more than Case III, and consequently, to an overall electricity cost that is around 320,000 DKK lower. In Figs. 7 and 8, the ranges of the overall flexibility that can be achieved for the two different types of RE are plotted. The results are obtained by running the simulations for 1000 different price sets considering Case I for the temperature setting.

From Figs. 7 and 8, we can see differences in the daily electricity consumption. This is due to the choice of the

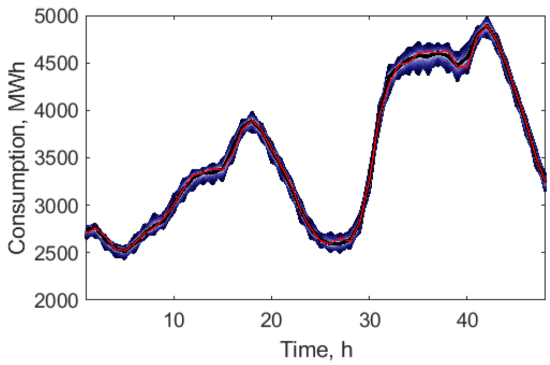

Fig. 7. Range of consumption achieved when considering static RE (i.e., model A). Base-line consumption (in black); Sample daily price response (in red).

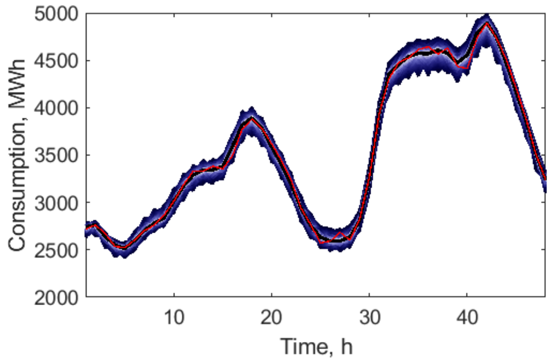

Fig. 8. Range of consumption achieved when considering dynamic RE (i.e., model B). Base-line consumption (in black); Sample daily price response (in red).

type of days represented, which are Sunday and Monday. The plots confirm the results given in Table 1, because Fig. 7 shows less flexibility in comparison with Fig. 8. For example, model A achieves a range of flexibility between 4.5 and 4.7 MW for hour 37, while this range is between 4.4 and 4.7 MW for model B (i.e., +50% than model A). Furthermore, when referring to the sample daily price response plotted in red, we can see that the total amount of flexibility provided by model A for up-regulation (or down-regulation, as the amount of regulation flexibility is the same for each flexibility direction α) is only 436 MWh, while model B provides 700 MWh. It also confirms that model B is able to achieve a higher amount of flexibility throughout the day.

In summary, it can be concluded that, when approaching model A for the entire pool of heterogeneous consumers, the operator might behave rather conservative in setting the dynamic prices. In fact, such a model overlooks a significant amount of the flexibility potential, which could be delivered between different R_j periods. Therefore, it is crucial for the operator to understand the dynamics of the flexible loads and combine the two RE models to be able to quantify the aggregate flexibility potential. Moreover, the operator needs to take into account the effect of the temperature on the overall price response of consumers, as this factor influences the overall results. If the dynamic prices submitted to the consumers are capped, the overall cost reduction might not be that significant for the consumers.

3.2 Modelling benchmark

Beside the different results in electricity cost and amount of flexibility provided, it is also interesting to investigate the computational performance of the two modelling approaches. In Table 2, we report the solution time, number of binary variables, MIP gap, number of equations and number of discrete variables for model A and B. From the table, we can conclude that model B takes longer to solve than model A. In our experimental setup, model A could be solved in less than 1 second on average, while model B required more then 82 seconds. The longer solution time can be explained by the larger amount of variables and equations in model B, in particular, the additional binary variables related to Eqs. (2a)-(2d). However, both models can be solved to optimality within a reasonable amount of time, which is indicated by the remaining MIP gap of 0.00 (i.e. less than the MIP gap tolerance of 10^{-5} set in the solver). We can conclude that the more flexible formulation of the RE requires some additional computational effort.

Table 2. Computational results of the two RE models (solution time (t); MIP gap (Gap); number of variables (#Var), number of binary variables (#Bin.V), number of equations (#Eq)

RE	t[s]	Gap[%]	#Var.	#Bin.V.	#Eq.
A	0.61	0.00	15402	8352	28451
B	82.41	0.00	19578	12528	33573

4. CONCLUSION

This paper investigates different approaches for modelling the RE of electrical consumers that respond to price-based DR programs. Two RE models are formulated as MILPs and applied to quantify the aggregate amount of flexibility that can be achieved when time-varying electricity prices are submitted to flexible consumers. In this study, the stochastic nature of consumers' behaviour toward prices is considered by approaching CC programming. The effect of the temperature is also investigated on the overall consumers' price response. Moreover, a computational study is provided for both models' performance, where the overall electricity cost of consumers and the amount of flexibility achieved by the operator are highlighted and compared for different RE models.

From the numerical results, it can be concluded that different RE models lead to significant changes in the overall flexibility potential. Therefore, it is crucial for the operators to have a deep understanding of the types of loads they deal with so that they can estimate the amount of flexibility more accurately. Introducing a specialised operator for the collection of off-line aggregate data can facilitate the understanding of consumers' dynamics. Alternatively, static RE can be considered for a more conservative study of RE. This approach avoids over-estimation of flexibility when loads' dynamics are difficult to estimate from aggregate measurements.

Due to the field data scarcity, we assume the condition of perfect RE in this study (i.e., where increase and decrease in consumption perfectly compensate each other). However, in future studies, an imperfect RE condition will be investigated for different consumers' categories.

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